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Can pipes be actually really that smooth?

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Abstract: In some recent papers a few approximations to the implicit Nikuradse–Prandtl–Karman equation were shown. The Nikuradse–Prandtl–Karman equation for calculation of the hydraulic friction factor is valid for the hydraulically smooth regime of turbulence. Accuracy of these approximations for the friction factor in so called smooth pipes is checked and related problems from the hydraulics are analyzed in the spotlight of the recently developed equations. It can be concluded that pipes can be treated as smooth below certain value of the Reynolds number but after that even new polished pipes with a minor roughness follow the transitional and subsequently the rough law of flow at a higher values of the Reynolds number.

Keywords: Accuracy; Comparison; Flow rate; Friction; Hydraulics; Piping

Mots clés: Precision; Comparaison; Debit; Frottement; Hydraulique; Tuyauterie

Nomenclature:

λ	Darcy (Darcy-Weisbach), i.e. Moody friction factor (-)
f	Fanning friction factor (-)
Re	Reynolds number (-)
ε	average height of protrusion of inner pipe surface (m)
D	inner diameter of pipe (m)
ε/D	relative roughness (-)
A, B, Co, ξ	auxiliary terms defined in the text

Abbreviations:

NPK Nikuradse–Prandtl–Karman

PNK Prandtl–Nikuradse–Karman

CPU Central Processor Unit

1. Introduction

Perfectly smooth surfaces do not exist (Taylor et al., 2006). Hydraulically smooth regime does not occur only in absence of the roughness (i.e. only when $\varepsilon/D=0$). This means that smooth regime can occur even if the relative roughness exists (if it is minor, i.e. if $\varepsilon/D\rightarrow 0$). This problem is shown in the spotlight of some recent new formulas.

2. Different hydraulic regimes

In their recent paper Li et al. (2011) analyze the flow friction factor with the special attention to so called "smooth" pipes. They note that the implicit equation developed by Colebrook (1939) is valid for rough pipes which should imply that its accuracy for "smooth" pipes can be disputed. The Colebrook equation is valid for the entire turbulent regime which includes the turbulent regime in the hydraulically smooth pipes, the transient (partially) turbulent regime and the fully turbulent regime in the hydraulically rough pipes. This is obvious from the title of the paper of Colebrook "Turbulent flow in pipes with particular reference to the transition region between the smooth and rough pipe laws". The Colebrook equation is not valid for the laminar regime which occurs for approximately $Re<2320$. It is valid for $2320<Re<10^8$ (the turbulent regime). It has to be noted that for the laminar regime, there are no smooth and rough pipes (Figure 1). Furthermore, in the laminar regime, all pipes are hydraulically smooth. If the pipe roughness (protrusions of inner pipe surface) is completely covered by the laminar sub-layer, the surface is smooth from the hydraulic point of view. In the laminar flow there is no laminar sub-layer, or better to say the main and only layer of flow is laminar, hence, the prefix ‘sub’ is sufficient (there is no turbulent layer). In other words, in the laminar regime, all pipes are “smooth” as mentioned before. With further increasing of the Reynolds number, thickness of the laminar sub-layer decreases baring the protrusions and fluid flow through a pipe becomes consequently hydraulically smooth, and then gradually roughs, both from the hydraulic point of view (Figure 1). Hence the introductory turbulent flow through the rough pipes (because the perfectly smooth pipes do not exist) can be noted as the hydraulically smooth. In the turbulent regime a rough pipe can be treated as smooth or rough which depends on the circumstances (Figure 1).

Figure 1. Different hydraulic regimes

Accuracy of the Colebrook equation can perhaps be disputed, but up to date it has been an accepted standard for the calculation of the friction factor in the turbulent flow both in, “smooth” and rough pipes. The well known Rouse and Moody diagrams (or better to say, their turbulent part) had been constructed using Colebrook’s formula (1):

$$\frac{1}{\sqrt{\lambda}} = -2 \cdot \log_{10}\left(\frac{2.51}{\mathrm{Re} \cdot \sqrt{\lambda}} + \frac{\varepsilon}{3.71 \cdot \mathrm{D}}\right) \tag{1}$$

For flow of natural gas or other gaseous fluids the coefficient 2.51 should be replaced with 2.825 according to the recommendation by AGA (American Gas Association) and American Bureau of Mines.

All equations in this short report are presented using the Darcy friction factor where the Darcy-Weisbach or the Moody friction factors are synonyms (here noted as λ). In the other hand, some researchers use the Fanning friction factor (f). This is also correct where the connection between these two factors is ($\lambda=4\cdot f$). The Fanning friction factor is C_f in Li et al. (2011).

3. Determination of hydraulic regime

As noted before, the turbulent regime can be divided into the three sub-regimes; i.e. in the "smooth" (introductory) turbulent regime, the partially (transient) turbulent regime and the rough (fully) turbulent regime.

The turbulent regime usually occurs when Re>2300 or slightly above. The Reynolds number was introduced by Reynolds (1883a,b) first in the "Proceedings of the Royal Society" followed by a longer paper in the "Philosophical Transactions of the Royal Society". Special issue of the "Philosophical Transactions of the Royal Society" dedicated to these papers was published in 2008 with title "Turbulence transition in pipe flow: 125th anniversary of the publication of Reynolds' paper". Further about the history of the Reynolds number can be seen in Rott (1990).

As shown in Brkić (2011a), the smooth regime of turbulence occurs only if $\xi<16$ and if Re>2320 while the rough turbulent regime occurs if $\xi>200$. Between is the transient (partial) turbulent regime ($16<\xi<200$). The Reynolds number (Re) is a well known parameter while a parameter ξ is defined by (3):

$$\xi=\frac{\varepsilon}{D}\cdot \mathrm{Re}\cdot\sqrt{\lambda} \qquad (3)$$

The value of parameter ξ as defined is valid for the Darcy friction factor (regarding the value of ξ for the Fanning friction factor readers can consult Abodolahi et al. (2007) where the hydraulically smooth regime occurs if $\xi_f<8$ and if Re>2320).

The NPK, i.e. the Nikuradse–Prandtl–Karman equation is Colebrook's equation in the total absence of roughness (when ε/D=0). The implicit NPK equation (2) cannot be derived from the Colebrook equation if the relative roughness is very small (ε/D→0). But hydraulically smooth regime also occurs in the technical systems when the relative roughness is significantly low (ε/D→0) and not only in the absence of roughness as can be seen from the figure 2 (the hydraulically smooth regime exist not only when ε/D=0 and Re>2300, but also when ξ<16 and Re>2300).

$$\frac{1}{\sqrt{\lambda}}=2\cdot\log_{10}\left(\frac{\mathrm{Re}\cdot\sqrt{\lambda}}{2.51}\right)=2\cdot\log_{10}\left(\mathrm{Re}\cdot\sqrt{\lambda}\right)-0.8 \qquad (2)$$

The NPK equation is actually the PNK (Prandtl-Nikuradse-Karman) in Li et al. (2011).

The Colebrook, as well as the NPK equation can be calculated only by using an iterative calculus or using approximate formulas. So, the main problem is not to find an approximate formula for the NPK equation which is valid only for the smooth part of turbulent regime, or better to say, for the turbulent regime in the absence of roughness (when ε/D=0). Problem is to find an approximate formula for the implicit Colebrook equation which is valid for the whole turbulent regime, including the smooth, transient and the rough portion of the turbulent regime.

Today, the NPK equation can be used only as an approximation for the Colebrook equation valid only for the "smooth" portion of the turbulent regime when the roughness can be neglected entirely (ε/D=0). This is a standard since 1939 when the paper of Colebrook was published and especially since 1944 when the paper of Moody was published (Moody, 1944).

4. Turbulent smooth regime

The NPK equation can be used only if $\xi<16$, calculated using the Darcy friction factor (and even then with disputed accuracy) and if Re>2300. As can be seen from figure 2, for $\varepsilon/D=0.01$, the upper limit for "smooth" regime is for Re≈6500 (the lower is 2300). This means that the NPK relation produces a relative error for this value of the relative roughness compared to the standard Colebrook equation of up to 24% (δ_1 from figure 2 with additionally $\Delta\delta_1$ compared with the equation for the hydraulically smooth regime by Buzzelli). Similar, for e.g. $\varepsilon/D=0.0005$, "smooth" regime is up to $Re\approx 2\cdot 10^5$ (and not below 2300) where the relative error compared to Colebrook is up to 17% (δ_2 from the figure 2 with additionally $\Delta\delta_2$ compared to the equation for the hydraulically smooth regime by Buzzelli). In theory, as relative roughness decreases ($\varepsilon/D\to 0$), the relative error also decreases rapidly ($\delta\to 0$).

Figure 2. Examined hydraulic problem

When the turbulent smooth regime is indicated, it is better to use an equation with the relative roughness included, such as those by Colebrook (1939) and Buzzelli (2008), and not those developed for use in the total absence of roughness. The bigger problem is how to measure or estimate the value of roughness accurately (Sletfjerding and Gudmundsson, 2003; Farshad et al., 2001).

4.1. Turbulent smooth regime in the absence of roughness

Li et al. (2011) have examined few equations valid for the "smooth" turbulent regime such as Blasius, Filonenko, etc (Table 1). Also, in their recent paper Danish et al. (2011) used the Adomian decomposition method (ADM) and the Restarted Adomian Decomposition Method (RADM) to develop their explicit approximation of the NPK equation (Table 1). Also Fang et al. (2011) have analyzed correlations of a single-phase friction factor for the turbulent pipe flow, also with the special attention to so called “smooth” pipes (Table 1).

Table 1: Equations for hydraulically smooth regime developed for the total absence of roughness

According to figure 2, equations proposed by Li et al. (2011), Danish et al. (2011), and Fang et al. (2011) can be used to substitute the implicit NPK equation very accurately (these are actually non-iterative explicit approximations to the implicit NPK equation). But also, they are very complex, hence, the equation by Filonenko or even simple power-law equations such Blasius can be used as an adequate and not that complex substitution to the implicit NPK equation.

Also as shown in Brkić (2011b), equations for the smooth regime in total absence of the roughness can be used as the base for the new approximations to the Colebrook equation if they are in suitable form for such transformation. Equations by Fang et al. (2011) and Li et al. (2011) are in such suitable form.

4.2. Turbulent smooth regime with presence of roughness

The equations by Buzzelli (2008) are very accurate explicit approximation of the Colebrook equation. Also, in addition, Buzzelli (2008) presented his equation specially developed for the “smooth” conditions (3):

$$\frac{1}{\sqrt{\lambda}} = A - \left(\frac{A + 2 \cdot \log_{10}\left(\frac{B}{\mathrm{Re}} \right)}{1 + \frac{2.18}{B}} \right) \qquad (3)$$

Where parameters A and B are defined as (3a,b):

$$A = \left(0.774 \cdot \ln(\mathrm{Re})\right) - 1.41 \qquad (3a)$$

$$B = \left(\frac{\mathrm{Re}}{3.7} \cdot \frac{\varepsilon}{D}\right) + 2.51 \cdot A \qquad (3b)$$

Approach by Buzzelli (2008) is good one because he uses the relative roughness even for the “smooth” regime of turbulence (Figure 2).

5. New equations developed at the Princeton and the Oregon University

According to Barenblatt et al. (1997) and Cipra (1996), the new reexaminations of classical and historically adopted relations for determination of hydraulic friction factor show that some of them are off by as much 65%. As noted in Cipra (1996), it seems that the many classical textbooks for hydraulics will have to be revised. For example, recent experiments at the Princeton University have revealed aspects of the smooth pipe flow behavior that suggest a more complex scaling than previously noted. The Princeton research shows that in the partially turbulent regime friction factor relationship follows an inflectional rather than the monotonic relationship given in the Moody diagram. Researchers from the Princeton concluded that friction factor behavior of a honed surface in the transitional regime does not follow Colebrook relationship and that for all conditions of roughness, logarithmic scaling was apparent at the higher Reynolds numbers with the same constants determined for smooth pipes. Another team at the Oregon University, working with a completely different type of facility have come to a similar conclusion. Note that the difference in scale of the Oregon and the Princeton devices is dramatic: for example, Princeton’s Superpipe weighs about 25 tons, whereas the Oregon tube weighs about 30 grams (McKeon et al., 2004). Another interesting conclusion from Cordero (2008) is that the power-law represents the velocity profile better than the logarithmic law for the Reynolds numbers below approximately 98 thousands.

Blasius's law (related to the power-law velocity profile) is considered more accurate than the NPK log-law in that region. From a practical point of view, it is best to apply Blasius's law up to Re=66,964 where it coincides with results by McKeon et al. (2004, 2005) thus providing a continuous friction factor.

McKeon et al. (2004) adjusted parameter in the NPK equation to fit the results obtained from the Princeton and the Oregon pipe facility (4):

$$\frac{1}{\sqrt{\lambda}} = 1.93 \cdot \log_{10}\left(\mathrm{Re} \cdot \sqrt{\lambda}\right) - 0.537 \qquad (4)$$

The NPK equation with the coefficients adjusted by McKeon et al. (2004) is also implicit in flow friction factor, hence the methodology proposed by Li et al. (2011) and Danish et al. (2011) can be applied to it. The Princeton and the Oregon equations for the fully developed pipe flow can be seen in McKeon et al. (2005). These equations for the fully developed pipe flow can be used as an improved substitution for the Colebrook equation valid for the whole regime of turbulence.

6. Comparison of different formulas

From figure 3 and 4, it can be seen that different equations produce the different results. But from figure 2, it also can be seen that these differences have a minor or none influence on the calculation. From figure 2, it can be clearly seen that only the effect of roughness can make an influence on the final results (to increase the accuracy of the final results).

Figure 3. Maximal relative error of presented equations for hydraulically smooth regime developed for the total absence of roughness

Figure 4. Comparison of the most accurate explicit approximations of the NPK equation

Same as the implicit Colebrook relation, its explicit approximations are valid for the whole turbulent regime. Up to date, only the approximation made by Churchill (1977) is valid for the turbulent and even the laminar flow of the Newtonian fluid (including the zone between them). According to the recent paper by Brkić (2011c), the approximations of the Colebrook equation by Romeo et al. (2002), Buzzelli (2008), Serghides (1984), Zigrang and Sylvester (1982) and Vatankhah and Kouchakzadeh (2008) are among the five most accurate up to date. Their relative error is no more than 0.15% compared to the iterative solution of the implicit Colebrook equation (for the whole turbulent regime). The other three approximations mentioned in the paper of Li et al. (2011) are not among the most accurate. These mentioned approximations are by Haaland (1983) with the relative error of no more than 1.5%, Swamee and Jain (1976) with the relative error of no more than 2.5% and Avci and Karagoz (2009) with the relative error up to 5%.

Measuring the CPU time is a good approach for a comparison of the formulas in hydraulics (Giustolisi et al., 2011; Danish et al., 2011; Li et al., 2011). Also, one has to be aware that computational speed does not depend only on the problem size but also on the computing environment (the type of CPU or other hardware components). Giustolisi et al. (2011) observe that the computation of logarithm in the computer languages is based on the series of expansions that require several powers of the argument to be computed and added to each other. Note that the explicit equations proposed by Danish et al. (2011) and Li et al. (2011) contain many logarithmic expressions. Approximations to the Colebrook equation examined by Brkić (2011c) are sorted based on their accuracy and the complexity as the criterions. The measurement of the CPU time can be a further step forward.

The Colebrook equation can also be transformed and approximately solved by using the Lambert W-function as shown in Brkić (2011d,e). The Lambert W function is also mentioned in Li et al. (2011).

7. Conclusion

It can be concluded that pipes can be treated as the smooth below certain value of the Reynolds number but after that even the new polished pipes with a minor roughness follow the transitional and subsequently the rough law of flow at the higher values of the Reynolds number. Today, the Colebrook equation is a standard for the calculation of flow friction factor with the particular reference to the transition region between the smooth and the rough pipe laws. It is implicit in the flow friction factor, but nowadays it can be solved easily using an iterative procedure or some of the very accurate approximations.

The Colebrook equation can be replaced with some of the new formulas like those developed from the data of the Princeton or Oregon pipe facility (Cordero, 2008), but the effect of the roughness cannot be neglected. Unavoidable the effect of roughness is the main reason why the Colebrook equation is a standard even for the smooth portion of turbulence. The roughness effect can be minor, but with slightly increased value of the Reynolds number, it will inevitable appear.

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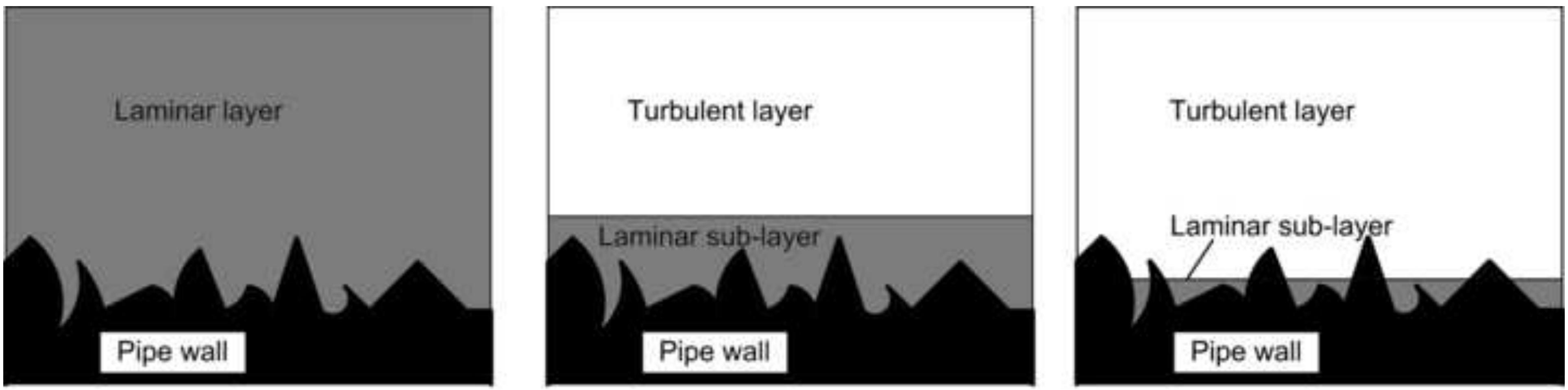

a) Laminar flow (smooth pipe) b) Hydraulically smooth pipe c) Hydraulically rough pipe

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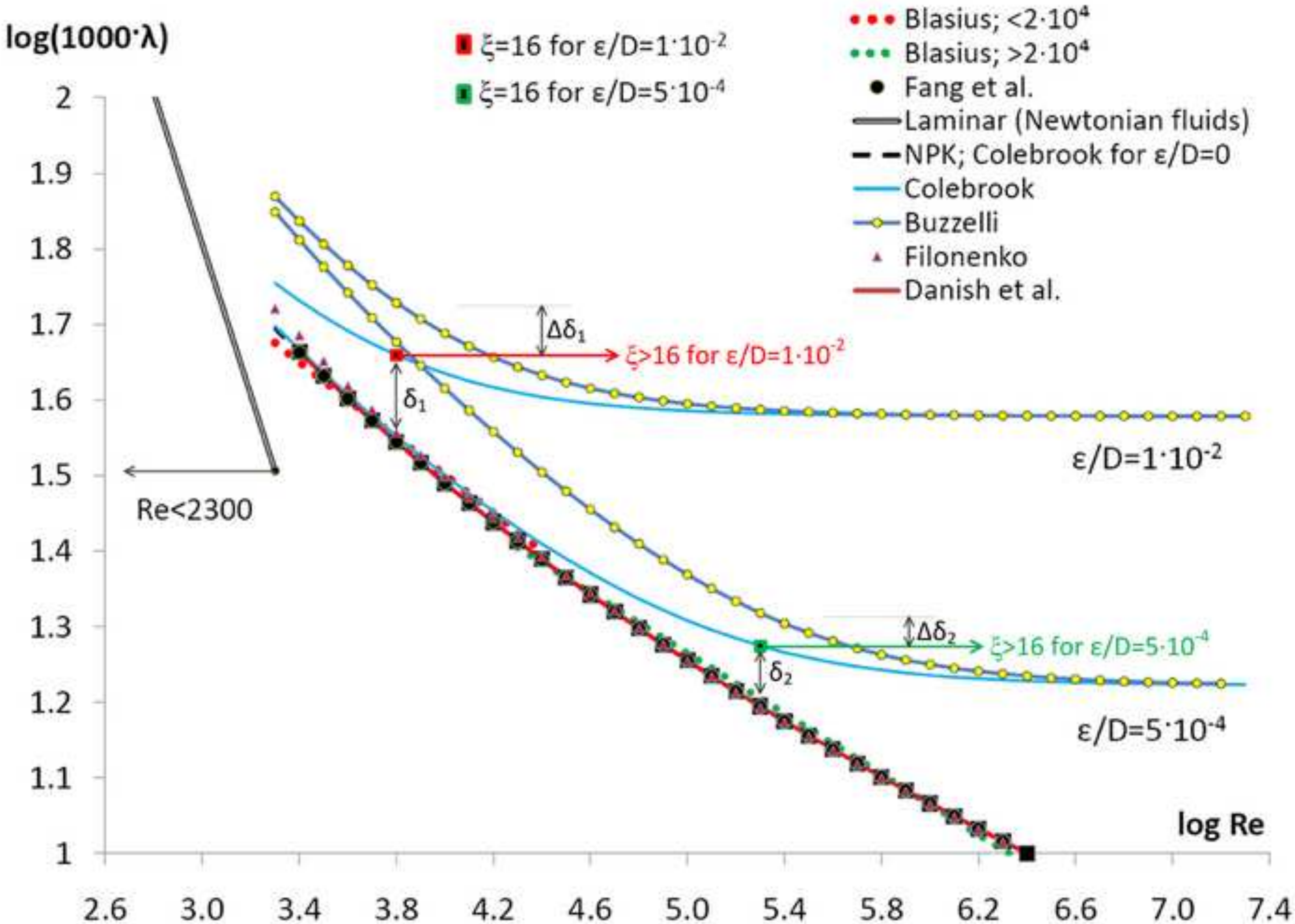

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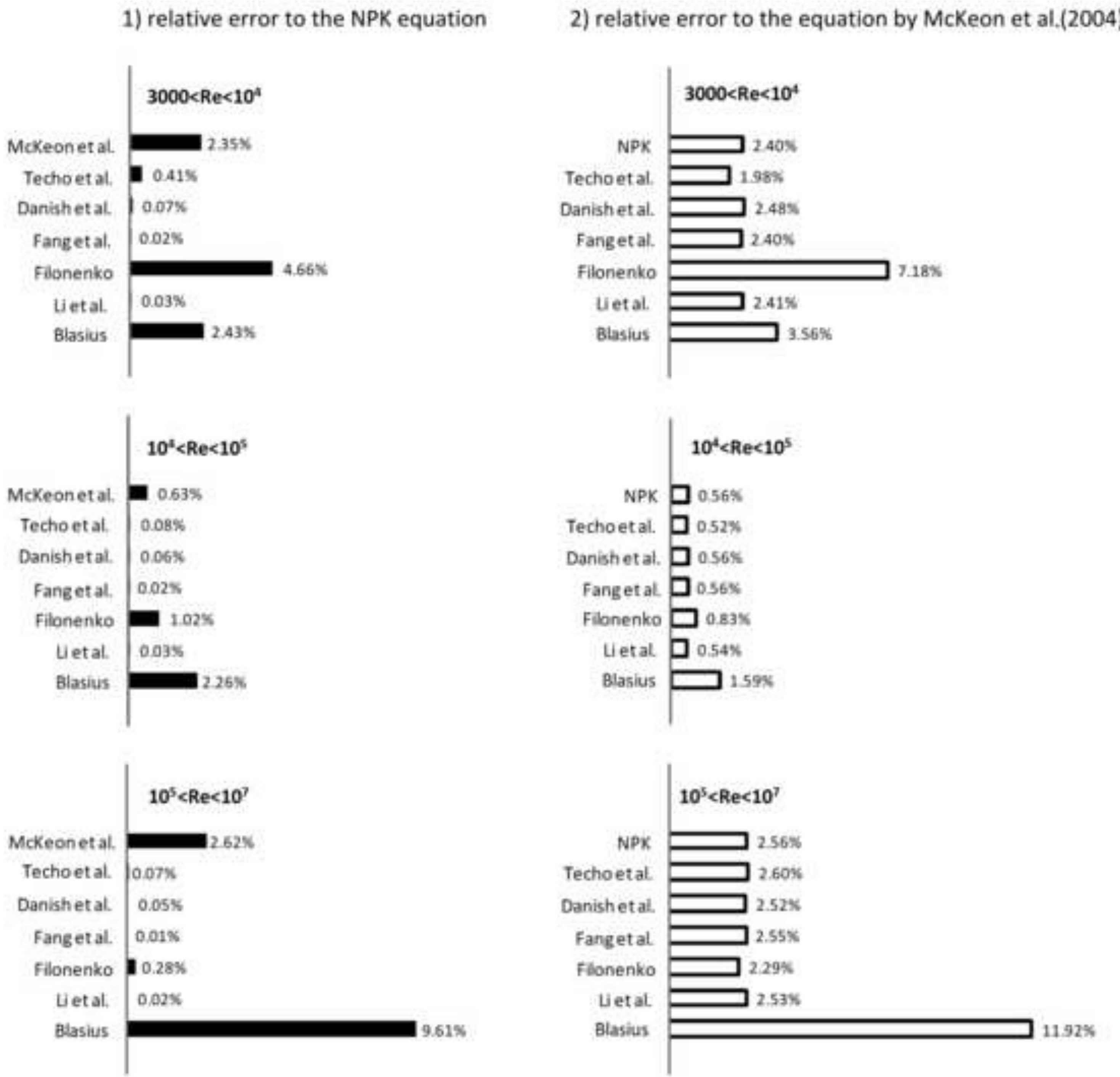

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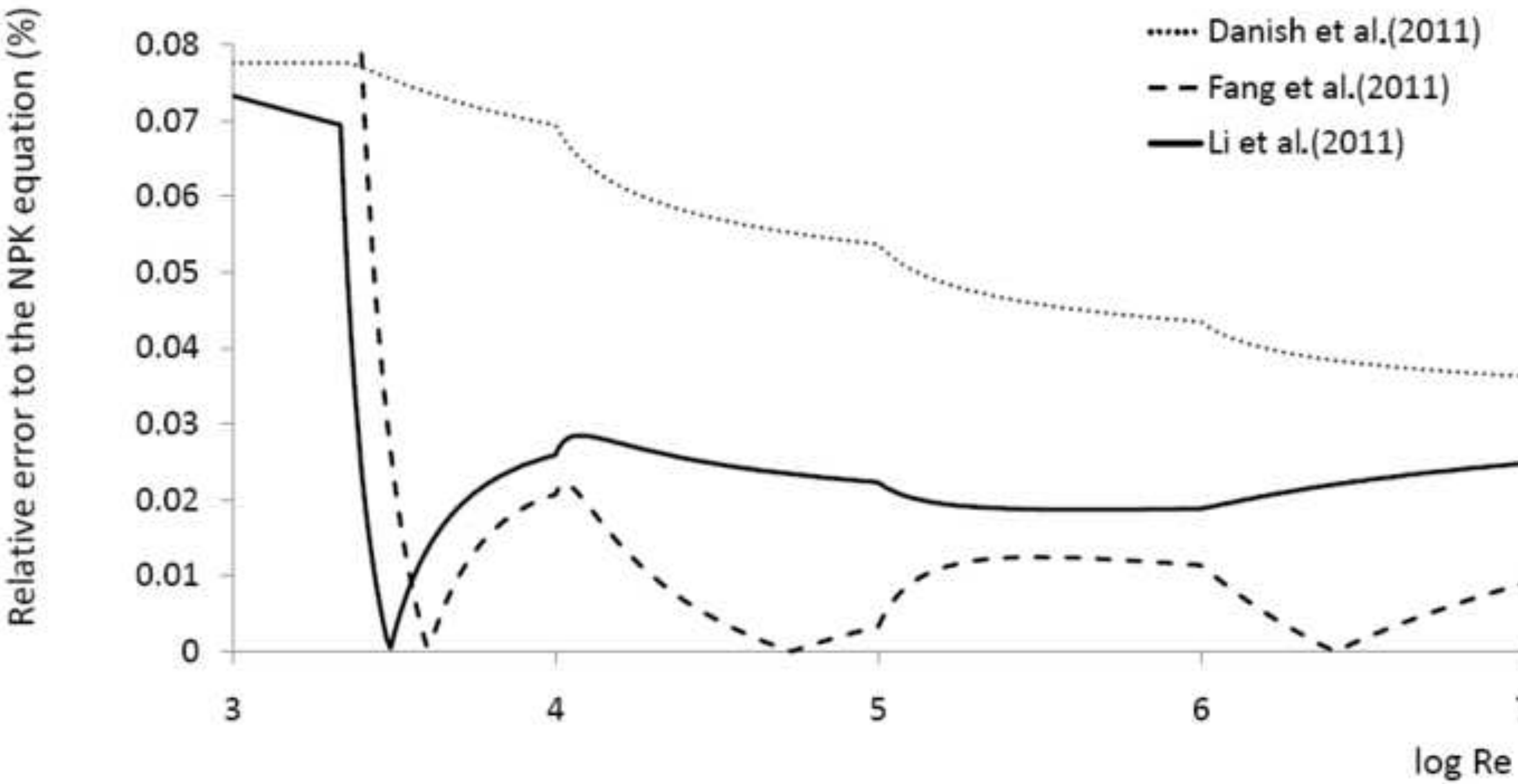

Table 1 DB

Table 1: Equations for hydraulically smooth regime developed for the total absence of roughness

Equation	Name
$\lambda = \dfrac{0.316}{\mathrm{Re}^{0.25}}$	Blasius (for Re<2·10^{4})
$\lambda = \dfrac{0.184}{\mathrm{Re}^{0.2}}$	Blasius (for Re>2·10^{4})
$\lambda = \left(0.79 \cdot \ln(\mathrm{Re}) - 1.64\right)^{-2}$	Filonenko
$\dfrac{1}{\sqrt{\lambda}} = 0.8686 \cdot \ln\left(\dfrac{\mathrm{Re}\cdot}{1.964 \cdot \ln(\mathrm{Re}) - 3.8215}\right)$	Techo et al.
$\dfrac{1}{\sqrt{\lambda}} = \dfrac{1}{2} \cdot \left(C_o - \dfrac{1.73718 \cdot C_o \cdot \ln(C_o)}{1.73718 + C_o} + \dfrac{2.62122 \cdot C_o \cdot [\ln(C_o)]^2}{(1.73718 + C_o)^3} + \dfrac{3.03568 \cdot C_o \cdot [\ln(C_o)]^3}{(1.73718 + C_o)^4}\right)$ $C_o = 4 \cdot \log_{10}(\mathrm{Re}) - 0.4$	Danish et al. (2011)
$\lambda = 0.25 \cdot \left[\log_{10}\left(\dfrac{150.39}{\mathrm{Re}^{0.98865}} - \dfrac{152.66}{\mathrm{Re}}\right)\right]^{-2}$	Fang et al. (2011)
$\dfrac{1}{\sqrt{\lambda}} = 0.8685 \cdot \ln\left(\mathrm{Re}\cdot \sqrt{\dfrac{-0.0015702}{\ln(\mathrm{Re})} + \dfrac{0.3942031}{\ln^2(\mathrm{Re})} + \dfrac{2.5341533}{\ln^3(\mathrm{Re})}}\right) - 0.198$	Li et al. (2011)[a]

[a]Also note that ln(Re)^2 and ln(Re)^3 are actually ln^2(Re) and ln^3(Re), respectively, i.e. (ln(Re))^2 and (ln(Re))^3. According to equation 7 in Li et al. (2011), one can assume that using logarithmic rule, ln(Re)^2 and ln(Re)^3 can be rearranged to 2·ln(Re) and 3·ln(Re), respectively which produce error (because 2 and 3 are power of whole logarithm and not only of Re)

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